EFFECT OF A JOINT ON THE MAGNETIC FIELD OF A LONG FERROMAGNETIC ROD

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In a number of applications it is necessary to employ long ferromagnetic rods (Fig. 1) which cannot always be made monolithic. This note examines the effect of an air gap on the magnetic flux through a ferrormagnetic rod and on the magnetic field of the rod.

When the ratio of the length of a cylindrical rod in a longitudinal homogeneous constant magnetic field to its diameter is large ($a/b > 5$), the circular cylinder can be replaced with a prolate ellipsoid of revolution.

To determine the effect of an air gap joint in the middle of an ellipsoid on its magnetic field by solving the corresponding boundary-value problem is a very complicated and laborious process. The formulas thus obtained would be clumsy and unsuitable for practical purposes.

Thus it is expedient to substitute for the constant external exciting field H_0 an electric current with linear density λ continuously distributed over the surface of the ellipsoid. The substitution should be made so that the magnetic field of the ellipsoid due to this current is equal to the magnetic field of the ellipsoid in the external field.

We obtain the linear current density λ by solving the corresponding problem of magnetostatics in the spheroidal coordinate system ξ, η, φ in the following form:

$$\lambda = H_0\left(1 - \frac{1}{\mu_r}\right) D \frac{\sqrt{1-\eta^2}}{\sqrt{(\alpha)^2 - \eta^2}}, \tag{1}$$

$$D = \frac{Q_1(\alpha) - \mu_r^{-1}(\alpha)}{\alpha^{-1}Q(\alpha) - \mu_r^{-1}Q_1'(\alpha)} \quad \left(\alpha = \frac{a}{q}\right), \quad q = \sqrt{a - b^2}.$$

Here μ_r is the relative permeability of the rod material; Q_1 and Q'_1 are a Legendre function of the second kind and its derivative for the argument indicated.

Integrating, we find the total equivalent current I

$$I = 2(1 - \mu_r^{-1})\, DqH_0. \tag{2}$$

After substituting the equivalent current for the external field, we consider the magnetic circuit obtained; we find [1] the total magnetic flux Φ_1 with the expression [1]

$$\Phi_1 = H_0(1 - \mu_r^{-1})\mu_0 AS, \tag{3}$$

$$A = -\frac{P_1(\alpha)\, Q_1'(\alpha)}{1 - P_1(\alpha)\, Q_1'(\alpha) / \mu_r Q_1(\alpha)} \frac{1}{Q_1(\alpha)}.$$

Here, S is the cross-sectional area of the ellipsoid and P_1 is a Legendre function of the first kind of the indicated argument.

We now find the total reluctance R_1 of the entire circuit without the air gap,

$$R_1 = \frac{I}{\Phi_1} = \frac{2Dq}{AS\mu_0}. \tag{4}$$

Here, μ_0 is the magnetic constant. The presence of an air gap causes an increase in the total reluctance by an amount R_2

$$R_2 = \frac{\Delta}{\mu_0 S}$$

(where Δ is the width of the air gap) and causes a decrease in total magnetic flux Φ_1 and, hence, in the external field of the ellipsoid, which can be characterized by the coefficient K,

$$K = \frac{\Phi_1}{\Phi_2} = \frac{R_1 + R_2}{R_1} = 1 + \frac{\Delta A}{2qD} = 1 + \left(\frac{1}{\mu_r} - \frac{Q_1(\alpha)}{Q_1'(\alpha)}\right)^{-1}$$

Here Φ_1 is the total magnetic flux without the gap and Φ_2 is the total magnetic flux with the gap.

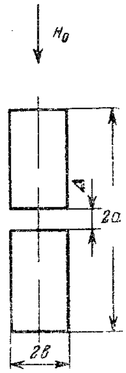

Fig. 1

Example. Determine the coefficient K for a centrally jointed rod made of steel with permeability $\mu_r = 185$ and with dimensional ratios $a/b = 10.1$ and $\Delta/b = 0.1$. In this case the coefficient K, calculated from (6), equals 1.18. The same quantity determined experimentally is equal to 1.13, indicating a discrepancy of 4.5%.

A comparison with the experimental data showed that (6) gives satisfactory results at $\Delta/b < 0.2$.

REFERENCE

1. N. N. Lebedev, Special Functions and Their Applications [in Russian], Fizmatgiz, 1963.

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